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An American Put Option Strategy for Real Asset Valuation: Predictive Decisions under a Monte Carlo Simulation

Steven Alan Lifland, Ph.D
Carl Maneval Smith Professor of Accounting and Finance
High Point University
High Point, NC, USA

Abstract

The income approach to asset valuation sets out to determine the expected future cash flows and discounts these cash inflows and outflows to find the aggregate net present value (NPV). Traditional capital budgeting assumes an equality of risk across investments and takes a static perspective of critical input variables. This paper extends the financial literature as it asserts that measuring the value of capital assets are significantly enhanced by the introduction of probability distributions as created through Monte Carlo Simulations. Ten input variables, subject to random movement, are ranked according to their impact on the project's NPV. Project acceptance/rejection is governed by both the NPV and MIRR calculations. Further, an American Put Option Strategy is applied, whereby management has the ability not the obligation to sell off the asset any time before the end of its investment horizon. Results revealed the possibility of a negative NPV existed for every year of the project's economic life. Traditional project analysis indicated that selling in years three through five would result in positive cash flows. Yet the distributions revealed probabilities of negative cash flows of 44%, 36% and 31% respectively.

Introduction

Budgeting decisions on capital assets, such as real estate, follow a uniform process. Incorporating local and regional property norms as well as analyzing competitive locations based upon the unique real estate properties themselves are common practice. In a broad perspective, the expected capitalization rates and the degree of risk associated with expected cash flows are of major concern. Recently, Lifland (2015) argued that the process of assessing the value of real estate properties, create real options that impact the value of the capital budgeting investment process. The creation of a financial put option on common stock and the reasoning behind it supports the creation of a real option in a real estate setting. It gives management the right but not the obligation to make a future sell decision. Just as with financial options, the value of the real option is contingent on future event(s) such as lease revenue receipts and the expected future re-sale or reversion value. When the exercise price exceeds the current value of the asset, Dixit and Pindyck (1994) argue that management will choose to exercise the option as it is perceived to be 'in-the-money'. Traditional capital budgeting analysis adheres to following NPV and IRR calculations. However, the results can create an illusion of certainty. One critical facet of the process, that is overlooked, is the possibility of negative future cash flows. If this occurs, the existence of multiple IRRs is assured. Static capital budgeting techniques do not adequately address the probability of the project incurring a negative net present value or the internal rate of return not exceeding the project's cost of capital. These techniques may try to address these concerns with the use of risk-adjusted-rates-of returns, to account for risk, but the interpretation of the results is still limited. This paper extends the literature by incorporating a Monte Carlo Simulation (MCS) that creates distributions for each of ten unique decision input variables that impact the annual net cash flows under an American Put Option Strategy.

Further, this paper reports the probabilities of projects achieving a desired outcome based on their net present values, profitability indexes, and modified internal rate of return (MIRR). This simulation ran fifty thousand probability scenarios (iterations), to produce probability distributions for all the critical input decision components as described in the property lease agreement.

This paper first looks at the related capital budgeting literature and the source of the data for the analysis. A discussion of the discounted cash flow (DCF) process with the resulting projected net cash flows from operations follows. Next, there is a detailed model of the MIRR making the case that it be considered as a supplementary measure in the capital budgeting process. The incorporation of the Monte Carlo methodology is reviewed and the probability distribution of the input decision variables under an American Put Option strategy is reported. The paper finishes with the analysis of the empirical results and the concluding remarks.

Related Literature

In a corporate capital budget analysis, the impact of risk and uncertainty on rational decision-making has been a major finance topic for discussion and research. The expected return in a capital budgeting case was found to be an increasing function of the risk-free rate of return, the market price of dollar risk, the project's variance of returns, the aggregate present value of the project and its co-variance with existing assets of the firm, and the co-variance of the project with other projects included in the capital budget (Litner, (1965)). Applying the Sharpe-Lintner-Black model of market equilibrium, the discounted cash flow models were found to depend on the periodic risk-adjusted discount rates. These rates, in turn, were adjusted for risk over the time periods as potential future cash flows were reassessed (Lucas and Prescott, 1971), (Fama, 1977), (Levy and Sarnat, 1984), and (Huang and Litzemberger, 1988). One weakness in the capital budgeting literature is that is generally accepted that investors will follow a set of rigid rules and will not alter a project at any specific stage of its useful operating life (Trigeorgis and Mason, 1987), (Trigeorgis, 1993).

An American Put Option in stock markets allow an investor to exercise the option to sell at any time before the expiration of the contract. The financial American put option implies that an early exercise ignores the value of waiting to see what the actual stock price will be at expiration. However, through an early exercise, the owner can capture the positive payoff position $(X - S) > 0$. Here it is the excess of the exercise price over the current stock price. This creates the ability to invest these excess funds over the remaining life of the put contract (Kolb and Overdahl, 2007).

Real options can exist in a capital budgeting framework allowing a strategic approach to decision making. Just as financial options derive their value from the underlying asset, the value of real options is contingent on future events (Xie, 2009). In a few hypothetical examples, the final decision on capital investments is influenced by future cash flows and discount rates but even more by the potential value that could be added from abandonment and end of period resale options (Bonini, 1977), (Berger, Ofek, Swary, 1996), and (Xie and Qi, 2008). Ignoring embedded options could result in underestimating the value of a project (Berger, Ofek, and Swary, 1995) and (Rose, 1998).

The implications are that management and investors gain a degree of flexibility through the recognition of the existence of real options and its impact on the valuation process. Traditional discounted cash flow (DCF) analysis settles project acceptance conflicts by deferring to the net present value (NPV) rule due to its focus on the cash flows of the project.

This paper extends the financial literature as it asserts that measuring the value of capital assets is significantly enhanced by the introduction of probability distributions as created through Monte Carlo Simulations. The resulting distributions reveal that adhering blindly to the NPV rule may prove to be problematic as the probability that the NPV could be negative existed in every year of the American Put Option strategy. The results imply that risk-averse investors can make choices, counter to the conventional NPV rule, that make financial sense.

Data Review

The financial facts and data for a commercial real estate property project were obtained from REIS, Inc. The company provides impartial commercial real estate performance data and analysis. It specifically focuses on the metro (city) and submarket (neighborhood) for the office, apartment, retail, and industrial sectors.

An actual commercial real estate property was analyzed as an independent project. The analysis is performed on a single downtown Chicago office building investment, 200 North LaSalle Street. It will be referred to as Project LaSalle. Initially, the data is incorporated into a traditional discounted cash flow (DCF) process whereby the primary output is the net present value, modified internal rate of return, and profit index. The physical characteristics for Project LaSalle are presented in Table 1 while the pertinent dollar per square foot data, used in the DCF model, is reported in Table 2. The net rentable area per square foot (psf) and the sale price (psf) are used to determine the initial outlay for the building. This is an historical purchase price from 2008 based on data from REIS, Inc.

Table 1
Physical Characteristics for Project LaSalle

Project Name	**Project LaSalle**
City	Chicago
Property Type	Multi-Tenant
Building Area (sf)	621,428
Building/Floors	1/30
Year Built/Renovated	1984/not yet

Table 2
Dollar Per Square Foot ($PSF) and Allocation Rate Data
Per Contract for Project LaSalle

	$psf	**Allocation Rate (%)**	
Net Rentable Area (sf)	621,428		
Sale Price psf	$ 175.00	Vacancy Loss	9.20%
Asking Rent psf	$ 26.36	Credit Loss	1.00%
Operating Expense psf	$ 11.89	Property Tax	10.00%
Expense Stop psf	$ 1 0.91	Insurance Expense	30.00%
Free Rent Concessions psf	$ 0.25	Maintenance Expense	60.00%
Capital Reserves psf	$ 0.11		
Going-In-Cap-Rate	7.50%		

Notes for Table Two's line items:

- All per square foot (psf) figures are on an annual basis.
- Net Rentable Area (NRA) of a building included in the transaction, expressed in square feet, is an approximation based on verified public records.
- The potential rent revenue is the product of the building rentable area estimate and the average asking rent which is the market rent paid by a potential tenant.
- Sale Price (psf) is the purchase price of the property per square foot of net rentable area (NRA).
- Asking Rent for office properties is a weighted average quoted as annual gross rent per square foot.
- Vacancy losses are estimated rent losses from unoccupied space and unpaid rents.
- The Expense Stop creates an upper limit on the amount of operating expenses that the owner will be responsible for.
- Expense Reimbursement Recovery is the difference between the operating expense psf and the expense stop psf. The excess must be paid by the tenant. The recoverable operating expenses are property taxes, insurance, and maintenance.
- Free Rent Concession, to induce the lease signing, is the offer of a free rent period during which no rent is required to be paid. It is the total dollar amount or number of months free rent granted per lease terms.
- Credit Loss is the total amount of rent due that the landlord is unable to collect due to tenant default.
- Operating Expenses are the average annual costs, per square foot, of operating buildings that include property taxes, energy, janitorial service, insurance, common area maintenance, and management and leasing fees.

- Capital Reserves is an allowance that provides the periodic replacement of building components that wear out more rapidly than the building itself. They must be replaced during the economic life of the building.
- The reported estimated Going-in Capitalization Rate (Cap Rate) can be compared to the Reis Indexed Metro Office Cap Rate of 7.4%. The REIS Indexed Metro Office Cap Rate is modeled as a function of risk-free interest rates, metro rent growth expectations, current construction activity, and by running measures of volatility in rents. These measures are proxies for capital conditions, income expectations, and risk.

Critical property benchmarks for the Chicago area were obtained from Metro Analysis and Rent and Sales Comparable reports supplied by REIS, Inc. and are presented in Table 3 below.

Table 3
Relevant Data and Risk Factors from Metro Area Analysis Conducted by REIS, INC.

	Chicago
Annualized 5-year Vacancy Rate	17.6%
Annualized 5-year Rent Growth	2.1%
Average Lease Term (years)	5.5
Average Leasing Commissions	4.1%
Annualized 5-year Construction/Absorption	1.9
Cap Rate	7.4%
Inflation Rate per www.InflationData.Com	3.85%

Notes:
Vacancy Rate is the amount of available space expressed as a percentage of total inventory.
Lease term is the average term currently being quoted for new leases, in years. This paper utilizes a 5-year lease life.
Leasing Commission is an amount paid to a real estate broker in exchange for bringing together the parties of the lease agreement.
Usually it's paid in the form of a percentage of the yearly rent.
Construction/Absorption is the construction or completions during the time period divided by absorption during the same time period.

The commercial property is acquired subject to existing leases as noted by the lease terms and leasing commissions in Table 3. All the components of Table 2 and the majority of those in Table 3 influenced the calculation of Project LaSalle's relevant future annual net operating income (NOI) and future reversion (RV) or sale price over the designated investment time horizon of five years.

Methodology

The data of Project LaSalle is incorporated in a traditional discounted cash flow analysis (DCF) method, resulting in the determination of critical output variables such as the annual net present value (NPV), modified internal rate of return (MIRR), and the profitability index (PI).

Analyzing projects under uncertainty is an attempt to find the parameters that influence the outcome of a particular project. It is often referred to as 'sensitivity analysis'. It is a process by which specific variables are modified in order to reflect an optimistic, most-likely, and pessimistic scenario. It seeks to find which variables impact an output (i.e., net present value) the most (Dayananda etal., 2008).

While this research conducts a sensitivity analysis regarding the Project LaSalle property, it pushes beyond the traditional discounted cash flow approach and the latter analysis. First, it utilizes the output metric of the modified internal rate of return as opposed to the more commonly used internal rate of return. The relevancy of the modified internal rate of return (MIRR) accrues from its overall strengths over the internal rate of return (IRR). Kierulff, (2008) argues the IRR can give an unrealistic view of a project's potential value. A project with positive and negative cash flows delivers multiple IRRs. It also ignores the firm's cost of capital. Regarding the existence of a real option, Plath and Kennedy (1994) state that both the future operating cash flows and the timing of the estimated future resale value allow the incorporation of relative risk which enables an investor to compare projects. Second, a full Monte Carlo Simulation is applied that creates probability distributions for ten unique input variables.

The results enable an investor to assess not only the subsequent impact on a designated parameter but to rank each input as to which ones cause the most dramatic shifts in said output. More meaningful than any traditional sensitivity analysis, the associated probability that the net present value metric will be positive and that the modified internal rate of return will exceed the project's capitalization rate are produced. The common approach to sensitivity analysis, while it recognizes the uncertainty of a project, comes up with only one case of each of the possible three situations. This paper ran 50,000 iterations for each of the input variables and the related outputs of the net present value, modified internal rate of return, and the profitability index. Further, an in-depth review of the project's cost-benefit tradeoff is done within an American Put Option framework. This involved running the iteration to determine the three parameters (NPV, MIRR, and PI) in each year of the five-year lease contract.

Discounted Cash Flow Method (DCF)

Under the discounted cash flow method (DCF), the expected future net operating income associated with the property is capitalized to determine the asset's estimated net present value (Gallinelli 2009). The DCF analysis helps to determine if a proposed project can generate strong enough risk-adjusted returns (DeLisle, 2009). The LaSalle project is reviewed under a standard framework for multi-period real estate investment analysis. There are changing rent rolls and lease renewals and lease variables (inflation and capitalization rate) that can change the level of net operating income (NOI), net terminal value or reversion value (RV) and net present valuation (NPV).

The basic DCF model to evaluate the property's net present value (NPV) is:

$$\text{NPV Office Building} = \sum NOI_t / (1+ capr)^t + RVt / (1+capr)^t - IO_0 \quad (1)$$

The NPV is equal to the present value of future cash inflows – initial investment.

where NOI = expected net operating income (cash flows) for the office building.

RV = reversion (resale) value of the property; net terminal value.

IO = initial investment outlay.

capr = Capitalization Rate for the office building.

t = unique time period for each of the expected future cash flows.

Table 4

Projected Net Cash Flows from Operations: Project LaSalle

Year	Factoid	2008	2009	2010	2011	2012
	1	2	3	4	5	
Rentable Area psf	621,428					
Average Asking Rate	3.85%	$26.36	$27.37	$28.43	$29.52	$30.66
Potential Rent Revenue	3.85%	$16,380,033	$17,010,664	$17,665,575	$18,345,699	$19,052,009
Vacancy Loss	9.20%	1,506,963	1,564,981	1,625,333	1,687,804	1,752,785
Effective Rent Revenue		$14,873,070	$15,445,683	$16,040,342	$16,657,895	$17,299,244
Operating Expense psf	3.85%	$11.89	$12.35	$12.82	$13.32	$13.83
Expense Stop psf	3.85%	10.91	11.33	11.77	112.22	12.69
Expense Reimbursement		$ .98	$1.02	$1.06	$1.10	$1.14
Expense Reimbursement		$608,999	$632,446	$656,795	$682,082	$708,342
Free Rent Concession	$.25	155,357	155.357	155,357	155,357	155,357
Credit Loss	1.00%	163,800	170,107	176,656	183,457	190,520
Effective Gross Revenue		$15,162,912	$15,752,665	$16,365,124	$17,001,163	$17,661,689
Total Operating Expenses	$11.89	$7,388,779	$7,673,272	$7,948,084	$8,243,605	$8,550,242
Capital Reserves	$.11	68,357	68.357	68,357	68,357	68,357
Total Expenses		$7,457,136	$7,731,629	$8,016,441	$8,311,962	$8,618.599
Net Cash Flow or (NOI)		$7,705,776	$8,021,036	$8,348,683	$8,689,201	$9,043,090

*Expected inflation/growth rate is 3.85%

**Other variable % and $ from Table 2

Notes for Table 4:

The estimated average annual inflation rate adjustment is 3.85%. The NOI increases each year even if leases are not renewed.
Vacancy losses are estimated rent losses from unoccupied space and unpaid rents.
Expense Stop creates an upper limit on the amount of operating expenses that the owner will be responsible for.
Expense Reimbursement Recovery is the difference between the operating expense psf and the expense stop psf. The excess must be paid by the tenant. The recoverable operating expenses are property taxes, insurance, and maintenance.
Free Rent Concession, to induce the lease signing, is the offer of a free rent period during which no rent is required to be paid. It is the total dollar amount or number of months free rent granted per lease terms.
Credit Loss is the total amount of rent due that the landlord is unable to collect due to tenant default.
Effective Gross Revenue is determined as the effective rent income plus the operating expense recoveries less the provisions for the free rent period and potential credit losses.
Operating Expenses are the average annual costs, per square foot, of operating buildings that include property taxes, energy, janitorial service, insurance, common area maintenance, and management and leasing fees.
Capital Reserves is an allowance that provides the periodic replacement of building components that wear out more rapidly than the building itself. They must be replaced during the economic life of the building.
Net operating income (NOI) is calculated as the net of the effective gross revenue and both the operating expenses and the provision for future capital outlays.
Even though the worksheet calculates the NOI, the measure is not income as described under generally accepted accounting principles (GAAP) but is cash flow. The term NOI is interchangeable with the net cash flow from operations.

This paper suggests that the appraised value of the project is a function of the income stream. It follows that the net operating income (NOI) cash flow results from the income stream that is generated from the operations of the property and that the real estate investment is independent of external factors such as taxes or financing. The investor is deciding upon a property's income potential not the property itself. The before-tax NOI serves as an objective means of measuring the potential income stream from the property while the going-in capitalization rate acts as an investor's subjective estimate of how well the capital is required to perform (Gallinelli, 2004). Tax benefits are not ignored, rather, the implication is that an investor will consider the before tax cash flows, understanding that a tax benefit will be realized. (Brueggeman and Fisher, 2008).

The expected returns for any particular group of investors should not be impacted by the financing of the project. It's not that interest rates or access to debt markets don't impact value, but under any economic climate, an investor will choose the equity-debt allocation based on the degree of risk that they are most comfortable with (Fisher 2008).

The Case For the Modified Internal Rate of Return (MIRR)

Both the net present value (NPV) and the internal rate of return (IRR) are accepted measures of analyzing the attractiveness of investments. However, there are two weaknesses of the IRR; the first being the assumption that future cash flows can be reinvested at the IRR; the second being that where future cash flows turn negative there is the existence of multiple IRRs. This results in an unrealistic view of a project's potential value (Kierulff, 2008). The inability of the IRR to deliver a single break-even point supports the use of the MIRR. Utilizing the modified internal rate of return (MIRR) results in a more conservative return than the IRR; negative cash flows are cancelled out by positive ones, and the cash flows are compounded forward at a more realistic reinvestment rate based on the project's capitalization rate. It then discounts future cash flows back to the initial outlay date at a rate that more fairly represents the investment risk of the project. The basic model to find the MIRR is presented below.

$$\text{Zero} = FVNOI_t / (1 + MIRR)^t + RV_t / (1 + MIRR)^t - IO_0 \quad (2)$$

The MIRR is the rate that equates the NPV to Zero
Future value of the sum of each NOI @ capr

$$\sum NOI_t (1+capr)^t = FVNOI \text{ at the end of the lease term} \quad (3)$$

where

RV = the reversion (sale) value at the end of the lease term
NOI = the net operating income or net cash flow for each year in the investment horizon.
capr = the capitalization rate used to determine the future value of net cash flows
FVNOI = the future value of the sum of each periodic NOI by the end of the lease term
RV = the Reversion value for the office building at the end of the lease term
MIRR = the modified internal rate of return for each office building
IO = the Initial investment outlay
t = the time period as of the end of the lease term.

In a work by (Lifland, 2015), a stratification of the MIRR was proposed in order to provide another layer of risk analysis. It was weighted by two major strata: the present value of the operating net cash flows and the present value of a project's terminal value. The assumption is that there is generally more certainty associated with earlier cash flows than the reversion value that occurs at the end of the investment horizon. With this in mind, the calculated weights of these cash flow streams on the MIRR reveal the relative risk associated with the return. The manager/investor can now see the sources of uncertainty in the valuation process that influences the decision to accept or reject a project.

Monte Carlo Simulation

The Monte Carlo Simulation is used to analyze models that contain uncertainty. It offers the ability to simulate a model so a variety of scenarios that might occur can be seen rather than a single best guess scenario. This is a perfect application for this LaSalle case where the supplied REIS, Inc. data gave such data points as an inflation/growth rate of 3.85% and a Going-in capitalization rate of 7.50%. There were ten metrics overall that could potentially change over time. Traditional analysis would treat them all as static over a fixed time horizon. Here, they are all modeled with probability distributions in order to see to what degree they may be considered as critical decision variables. These probability distributions produce parameters such as the mean, minimum, maximum, and standard deviation of a normal distribution along with the determination of a lower 5% percentile and upper 95% percentile. In the running of the simulation, there is a recalculation based on fifty thousand iterations. After each iteration, a sample random variable is generated for each decision variable containing the probability distribution. For any stochastic model, the first requirement is the ability to generate random variables. The reproduction of a sequence of random numbers is important for reducing the variance of the distribution (McLeish, 2005). The analysis is performed in Excel and makes use of @RISK software. There are two key functions. The first is the =NORMINV(Rand(), mean, standard_dev) which returns the inverse of the normal cumulative distribution for the specified mean and standard deviation. It uses an iterative search technique. Within this function, the Rand() function returns a random number greater than or equal to zero and less than 1, evenly distributed and changed on recalculation. It takes the cumulative probability as input and provides the value of the decision variable corresponding to that cumulative probability. This paper looks at a 'Full' Monte Carlo Simulation as it includes ten unique decision input cash flow variables found in the Discounted Cash Flow Analysis and are listed in Table 5 below:

Table 5
Project LaSalle
Input Decision Variables in the DCF Analysis
Utilized in a Monte Carlo Simulation
Under an American Put Option Strategy

1. Asking Price of the Property per square foot (psf)
2. Expense Stop per square foot (psf)
3. Operating Expenses per square foot (psf)
4. Credit Loss, a percentage of the potential rent revenue
5. Insurance Expense, a weighted allocation of Operating Expenses
6. Maintenance Expense, a weighted allocation of Operating Expenses
7. Property Tax, a weighted allocation of Operating Expenses
8. Selling Costs, a percentage of gross reversion value
9. Vacancy Loss, a percentage of the potential rent revenue
10. Going-In Capitalization Rate, the initial cost of capital subject to future volatility

These decision variables directly impact:

11. Present value interest factors (PVIF) leading to net present value (NPV)
12. Future value interest factors (FVIF) leading to Modified Internal Rate of Return (MIRR)

The probability distributions of the ten decision variables, including the capitalization rate, for Project LaSalle, are presented in Table 6 below. The motivation for this full Monte Carlo Simulation will be evident as the Discounted Cash Flow Analysis is no longer dependent on single static decision variables but now incorporates a stochastic distribution of values.

Table 6
Project LaSalle
Probability Distribution of the Input Decision Variables
Under an American Put Option Strategy

Decision Input Variable	Min	Mean	Max	Lower 5% Percentile	Upper 95% Percentile
Asking Price	-0.53%	3.85%	8.15%	2.21%	5.49%
Expense Stop	-0.27%	3.85%	8.10%	2.21%	5.49%
Operating Expense	-0.43%	3.85%	8.60%	2.21%	5.49%
Credit Loss	-3.53%	1.00%	5.85%	-0.64%	2.64%
Insurance Expense	-0.63%	3.85%	8.23%	2.21%	5.49%
Maintenance Expense	-0.59%	3.85%	8.31%	2.21%	5.49%
Property Tax	-1.70%	2.50%	6.65%	0.86%	4.14%
Selling Costs	-1.19%	3.00%	7.66%	1.36%	4.64%
Vacancy Loss	4.86%	9.20%	13.34%	7.56%	10.84%
Capitalization Rate	3.35%	7.50%	12.03%	5.86%	9.14%

Empirical Results

Under the American Put Option strategy, an investor can approach the valuation process where the property can be considered for sale at the end of any year within the investment horizon and not just the end of the lease term. The reality is that a reversion or sale value must be determined for each annual future cash flow under consideration. The reversion value (RV) is calculated by dividing the future net operating income (NOI) by the given property's estimated going-in capitalization rate (Cap Rate). The expected future reversion values from the resale of the property under an American Put Option Strategy are presented in Table 7 below.

Table 7
Project LaSalle
Projected Reversion Values for each Year in the Lease Term
Under an American Put Option Strategy

Year of Cash Flow	Year of NOI	NOI	Capitalization Rate	Annual Gross Reversion Value	Selling Costs	Net Reversion Value
1	2	$8,021,036	7.50%	$106,947,151	3.00%	$103,738,736
2	3	$8,348,683	7.50%	$111,315,779	3.00%	$107,976,305
3	4	$8,689,201	7.50%	$115,856,007	3.00%	$112,380,327
4	5	$9.043,090	7.50%	$120,574,528	3.00%	$116,957,292
5	6	$9,410,872	7.50%	$125,478,292	3.00%	$121,713,943

Note: American Put Option – periodic NPV based on unique reversion value at the end of each period.
Selling Costs start out as 3% of the Gross Reversion but will change over time due to Monte Carlo Distribution.
Capitalization Rate starts out as 7.50% but will change over time due to Monte Carlo Distribution.
The final net reversion value shown in this table is based on a static capitalization rate of 7.50% and Selling cost rate of 3.00%.
Year 3: NOI4/Cap rate = $8,689,201/.075 = $115,856,007 less 3% selling cost = $112,380,327

The cost of Project LaSalle is $175.00 per square foot and the office building has rentable square footage of 621,428. The building is thirty floors high. This translates into an initial cost of $108,749,900. Within Table 8 below, the individual unique net cash flows are reported for each year of the lease along with the inclusion of the initial cost of the property. In addition, there are the financial metrics of the net present value (NPV), profit Index (PI), and the modified Internal Rate of Return (MIRR). Two critical outputs of the Monte Carlo Distributions, the probability that the NPV is less than zero and the probability that the MIRR exceeds the going in capitalization rate, are also generated. These latter two statistics foreshadow the results of the detailed probability distributions as reported in Table 9.

Table 8
Project LaSalle
Discounted Cash Flow Analysis with NPV, PI, and MIRR
Under an American Put Option Strategy

	0	**1**	**2**	**3**	**4**	**5**
YEAR 1						
Net Cash Flows	$(108,749,900)	$ 111,444,512				
NPV	$(5,080,586)					
Profit Index	.953					
MIRR	2.48%					
Prob. NPV < 0	63.14%					
Prob. MIRR > Cap	36.05%					
YEAR 2						
Net Cash Flows	$(108,749,900)	$ 7,705,776	$ 115,997,342			
NPV	$(1,205,453)					
Profit Index	.989					
MIRR	6.90%					
Prob. NPV < 0	53.17%					
Prob. MIRR > Cap	46.53%					
	0	**1**	**2**	**3**	**4**	**5**
YEAR 3						
Net Cash Flows	$(108,749,900)	$ 7,705,776	$ 8,021,036	$ 120,729,010		
NPV	$ 2,541,221					
Profit Index	1.023					
MIRR	8.33%					
Prob. NPV < 0	43.90%					
Prob. MIRR > Cap	57.50%					
YEAR 4						
Net Cash Flows	$(108,749,900)	$ 7,705,776	$ 8,021,036	$ 8,348,683	$ 125,646,492	
NPV	$ 6,163,649					
Profit Index	1.057					
MIRR	8.99%					
Prob. NPV < 0	36.40%					
Prob. MIRR > Cap	67.08%					
YEAR 5						
Net Cash Flows	$(108,749,900)	$ 7,705,776	$ 8,021,036	$ 8,348,683	$ 8,689,201	$ 130,757,033
NPV	$ 9,665,907					
Profit Index	1.089					
MIRR	9.35%					
Prob. NPV < 0	30.58%					
Prob. MIRR > Cap	74.49%					

This table presents the NPV for each year of the five-year lease for Project LaSalle. Adhering to the American Put Option Strategy suggests that management has the opportunity to sell off the property any time before the expiration of the property lease term. The figures presented here in Table 8 were generated based on the given static mean decision parameters depicted in Figure 1. According to traditional DCF analysis and the Put Option, the property will only be elected to be sold off in years three, four, and five. The results of the distribution strongly supports the negative NPV with probabilities of 63.14% and 53.17% in years one and two, respectively. However, while year three displays a positive NPV of $2,541,221, the Monte Carlo distribution reveals an approximate 44% chance that it could have been negative which implies that accepting the decision to sell off at year three could prove to be a losing proposition. Even the MIRR reveals that there is little more than a fifty-fifty chance that it will exceed the capitalization rate. In year 4 the NPV improves to a positive $6,163,649 with a stronger MIRR of approximately 9.0% but even in this scenario, there is still a 36.4% probability that a negative NPV can happen. The final year of the lease reveals a strong chance (74.5%) that the MIRR is greater than the Capitalization Rate and a lower chance of negative cash flows (30.58%). The results are further disseminated in Table 9 and Table 10 that report the detailed Monte Carlo distribution results for both the NPV and MIRR.

Table 9
Project LaSalle
Monte Carlo Simulation Results for Individual NPVs
Under an American Put Option Strategy

Annual NPV	Minimum	Mean	Maximum	Lower 5% Percentile	Upper 95% Percentile
NPV 1	$(44,844,970)	$(3,129,036)	$126,024,600	$(24,600,660)	$24,931,480
NPV 2	(43,391,220)	832,830	138,328,300	(22,186,000)	30,857,890
NPV 3	(46,075,770)	4,677,980	153,719,200	(20,420,390)	37,117,260
NPV 4	(51,217,560)	8,410,455	169,678,700	(19,036,020)	43,808,550
NPV 5	(56,207,530)	12,034,150	186,017,900	(17,984,000)	50,593,230

TABLE 10
Project LaSalle
Monte Carlo Simulation Results for Individual MIRRs
Under an American Put Option Strategy

Annual MIRR	Mean	Lower 5% percentile	Upper 95% Percentile
MIRR 1	4.27%	(15.60%)	30.22%
MIRR 2	7.55%	(2.77%)	20.05%
MIRR 3	8.68%	1.60%	16.94%
MIRR 4	9.21%	3.66%	15.45%
MIRR 5	9.49%	4.78%	14,61%

Note: The going-in cap rate for Project LaSalle is 7.50%

Both Tables 9 and 10, as created by a Monte Carlo Simulation, corroborate the premise of this paper that asset valuation analysis is significantly enhanced by the introduction of probability distributions. Specifically, there is a relatively strong chance that a static analysis could lead to making an incorrect decision. It was shown in Table 7, that a traditional DCF analysis, characterized by static input variables, that there were definite rejections of the notion to sell the project in years one and two and that in the following years of three, four, and five, it reports definite acceptances of the decision to sell off the capital asset. A critical result from the probability distributions analysis was that in every year of the proposed American Put Option Strategy, there existed a possible negative NPV. In the case of the MIRR, focusing on the Lower 5% Percentile column in Table 10, the MIRR is consistently below the Going-In Capitalization Rate for every year in the investment horizon. Even if one argues for the introduction of scenario analysis in the capital budgeting process, misleading decision criteria can result as scenario evaluations usually involve only three or four unique iterations. Contrast that with this paper's results that focused on ten unique decision input variables after 50,000 iterations were conducted.

Obtaining actual key component data for the property and using it in the capital budgeting process is an important feature of this work. While actual data is also meaningful under common budgeting analysis, the fact that it would be treated as static, could bias the results and make them less reliable. This paper measures the volatility of each of the ten input variables found in Table 6 and then estimates the dollar impact of each input variable on the NPV in each of the years of the five-year lease term for Project LaSalle. An interesting and critical result is the ranking of each input, from highest to lowest influence on the NPV. Further, the pattern of these rankings over time reveal which are the most significant and least significant of the input variables. These results are presented in Table 11 below:

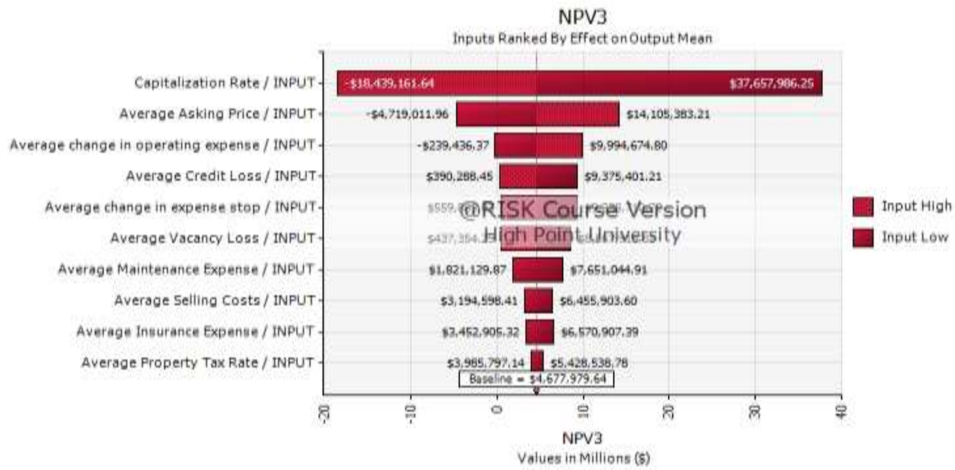
Table 11
Project LaSalle
Mean Net Range Impact of Inputs on NPV Outputs
Inputs Ranked by Their Effect on NPV Outputs
Under an American Put Option Strategy

Inputs Impacting the NPV	NPV 1 Mean Net Range	NPV 1 Rank of Inputs	NPV 2 Mean Net Range	NPV 2 Rank of Inputs	NPV 3 Mean Net Range	NPV 3 Rank of Inputs	NPV 4 Mean Net Range	NPV4 Rank of Inputs	NPV 5 Mean Net Range	NPV 5 Rank of Inputs
Cap Rate	$51,934,734	1	$53,982,379	1	$56,097,148	1	$58,273,806	1	$60,507,421	1
Asking Price	5,998,652	4	12,398,173	2	18,824,395	2	25,271,341	2	31,733,445	2
Operating Expense	3,761,624	5	6,990,890	5	10,234,111	3	13,488,245	3	16,750,458	3
Credit Loss	8,245,128	2	8,618,145	3	8,985,113	4	9,346,168	5	9,701,444	5
Expense Stop	2,780,937	7	5,773,281	6	8,778,847	5	11,794,855	4	14,818,715	4
Vacancy Loss	7,551,823	3	7,845,420	4	8,130,565	6	8,407,532	6	8,676,590	7
Maintenance	2,022,349	8	3,912,372	7	5,829,915	7	7,754,845	7	9,685,482	6
Selling Costs	3,436,987	6	3,347,168	8	3,261,305	8	3,179,266	8	3,100,924	9
Insurance	1,341,439	9	2,105,150	9	3,118,002	9	4,134,322	9	5,153,181	8
Property Tax	832,492	10	1,107,440	10	1,442,742	10	1,774,202	10	2,101,823	10

The sensitivity analysis presented in Table 11 needs further clarification. These results are taken from what is referred to as tornado diagrams or charts. Their purpose is to compare the relative importance or impact of specific items on an output such as the net present value. In essence, the input variables are modeled as uncertain values. This paper tested for the sensitivity associated with each input variable. While Table 11 allows us to visualize the net impact of ten different variables, the mean net range incorporates both the low and high impact into one number. Refer to the year three tornado chart (Figure 1) under the American Put Option Strategy. There are three major points to understand: (1) the longer the bar the stronger the sensitivity of the NPV to the factor, (2) there is a top-down priority given to the factor with the largest impact, and (3) the ends of the bar reflect the low and high value of the input factor. It is this last fact that is not apparent in Table 11 above. In year three of the strategy, the capitalization rate has the greatest impact with a low end of -$18,439,162 and a high end of $37,657,986. The mean net range reported in Table 11 is calculated as the [high end value – low end value] [37,657,986 – (18,439,162) which results in the amount of $56,097,148. The second highest impact variable was the asking price rent which had a low end value of -$4,719,012 and a high end value of $14,105,383 resulting in a mean net range of 18,824,395. One of the contributions of this paper is that these results allow management to realize the extent to which the annual NPV is impacted by the uncertainties of specific input variables.

A trend emerges from the data in Table 11, where the capitalization rate and asking price on the lease were consistently ranked one and two from years two through five of the Put Strategy. The dollar figures further suggest that the annual NPVs were extremely sensitive to changes in these two factors. The going-in cap rate had the highest impact on the NPV in each year from time period two through time period five. The net dollar impact ranged from $51.9 million in year one to $60.5 million in year five. Contrast this with the least impactful parameter, property taxes, which ranged from $ .832 million to $2.10 million over the same period. The selling costs, insurance expense, and property taxes were ranked at the bottom in four of the five years. The credit loss, expense stop, vacancy loss, and maintenance were mixed in their impact as they changed places among the four, five, six, and seven slots. The trend and impact of the expense stop is one of a consistent increase from being ranked seventh in year one, sixth in year two, fifth in year five, and fourth in years four and five. This trend increased the future cash flows of the project. In year one of the strategy, where a high probability of a negative NPV exists, credit loss and vacancy loss were ranked two and three, respectively. These appear to be major factors in reducing the projected cash flows in year one which did not help the NPV situation. By year five, the impact of vacancy loss had slipped to seventh place while the impact of credit loss stood in fifth place.

Figure 1
Project LaSalle
Tornado Chart in Year Three
Inputs Ranked by Impact on the NPV



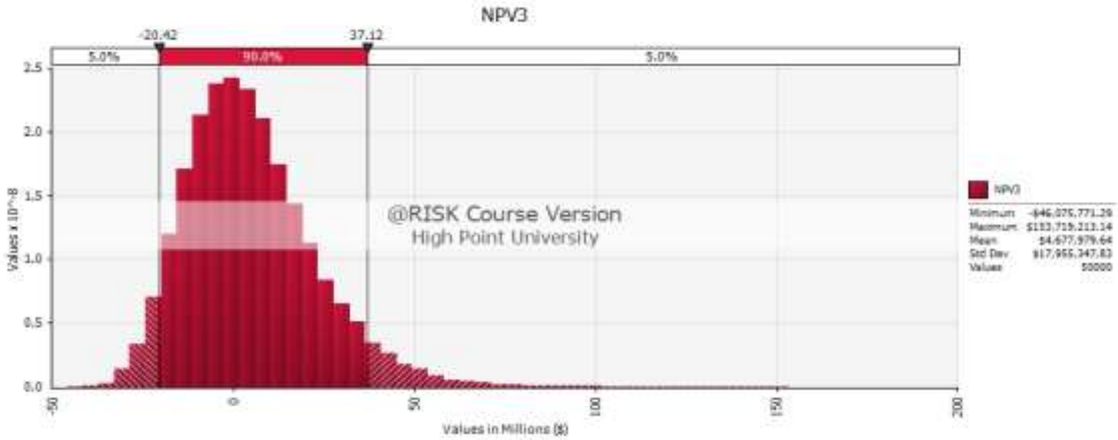
NOTE**:** Attributes of the Tornado Chart

[1] The longer the bar, the greater the sensitivity of Project LaSalle's NPV to a specific factor
[2] The factor or input that has the greatest impact is located at the top; i.e., Capitalization Rate
[3] The ends of the bar indicate the low and high value of the input factor
[4] Measuring the breadth of each bar equals the Mean Net Range in Table 11. For example, the average credit loss ranged from a low $390,288 to a high of $9,375,401 in Figure 1. It is reported in Table 11 as a mean net range of $8,985,113 ($9,375,401 – 390,288).
[5] Tornado charts were run for each year but only Year Three is shown here for explanatory purposes.

Defining the Probability Distribution of the Annual NPV

The Monte Carlo Simulation allows the fitting of probability distributions to the Project LaSalle data. Actual historical data concerning the critical square footage components of this Chicago office building has been collected and distributions of possible future annual net present values were created. When the simulation is run, financial models are calculated repeatedly. Each iteration represents a set of new values derived from each of the inputs used in the DCF analysis and a new possible NPV is generated as an output. New possible outcomes are created from each iteration. Figure 2 reports the graph of the distribution of possible NPV outcomes in year three. For this simulation, there were 50,000 iterations conducted. The graph reflects statistics on how the NPVs are distributed across their minimum and maximum range. The probability distribution of the input decision variables is reported in Table 6 above. This identified the input distributions most critical to the NPV results. A histogram of the possible NPVs for year three of the American Put Option Strategy is presented below.

Figure 2
Project LaSalle
Histogram of Possible NPV's with Positive & Negative Correlations
Year Three Under an American Put Option Strategy

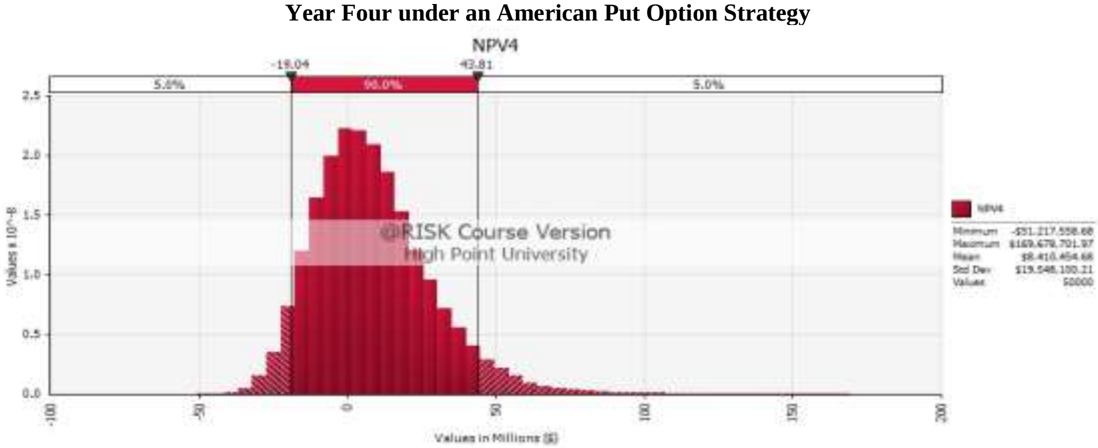


NPV Profile histograms were run for each year but only Year Three is shown here for explanatory purposes.

Within Table 8, year three is the first time period, under the American Put Option Strategy, that has a positive NPV and that value was $2,541,221. Traditional capital budgeting rules suggest projects are accepted where the NPV is greater than zero, not a negative value. The results of this study show this may be an ill-informed commitment. The histogram (Figure 2) reveals that while the mean NPV is approximately $4,677,980 and the maximum value can be $153,719,213, there exists the possibility of a minimum value of a negative $46,075,771 value. There is an approximate 44% probability of that negative NPV occurring (Table 8). This paper posits that this cannot be ignored in evaluating Project LaSalle. While 90% of the distribution is scattered about the mean value, the horizontal axes (the lower 5th and upper 95th percentiles address the issue of volatility. The standard deviation, a measure of risk and volatility, was approximately $17,955,348.

Contrast the Year Three graph with the histogram of possible NPVs for Year Four under an American Put Option strategy by viewing Figure 3 below.

Figure 3
Project LaSalle
Histogram of Possible NPV's with Positive and Negative Correlations
Year Four under an American Put Option Strategy



NPV Profile histograms were run for each year but only Year Four is shown here for explanatory purposes.

Within Table 8, the NPV at the end of period four was a positive $6,163,649 indicating that year four would be an acceptable year in which to sell off the property and earn a positive cash flow. The histogram reveals a positive mean NPV of approximately $8,548,100. However, just as was seen in Year 3, the minimum is a negative value. In fact, the negative value of approximately $51,217,559 is slightly higher than found in year 3.

The fact was that a possible negative NPV was found in each of the five years of the American Put Option Strategy. The probability of a negative NPV was 36.4%. If you compare the horizontal axes of these two years, we find that the net range for year three was 57.54% while the net range for year four was 62.85%. This supports the notion that the further out cash flows are promised, the more risk is associated with those net cash flows. This result also addresses the concern of prior work by (Lifland, 2015) that suggested the stratification of the MIRR, in order to give a weight to the riskier future cash flows, operational and terminal, upon which projects rely to arrive at a decision to accept or reject.

Review of Key Empirical Results

The probabilities of the behavior of both the annual NPV and MIRR, along with their mean values, over the five-year life under an American Put Option Strategy is shown in Table 12 below. The striking results are that each of the annual mean NPVs while positive in four of the five years of the American Put Option Strategy, the lower 5% percentile is negative in every case. The probability that the target NPV output is negative declines from a high of 63.14% in year one to a relatively lower probability of 30.58% by the end of year 5. It should not be lost on the analysis of Project LaSalle, that despite including all five years of net cash flows into the model, there was still roughly a 31% chance that the property would not be able to be sold off where the NPV was positive. The MIRR shows an expected consistent increase in the probability that it would exceed the average cap rate of the project. In year one, there is only a 36.05% chance of this occurring. By the end of year five, there is a 74.49% probability that the MIRR would be greater than the capitalization rate. The lower 5% percentile shows, in years three through five, that while the MIRR declines below the mean capitalization rate, it never turned negative. Within the lower 5% percentile, the MIRR never exceeded the property's cap rate. The upper 95% percentile, not shown in Table 12, is more robust as the MIRR exceeds the mean rate in each of the five years of the American Put Option Strategy.

Table 12
Project LaSalle
Probabilities of both the Annual NPV and Annual MIRR
Over the Five Year Term of an American Put Option Strategy

Year	Mean NPV	5% Percentile	Probability NPV < 0	Mean MIRR	5% Percentile	Probability MIRR > Cap Rate
1	$(44,844,970)	$(24,600,660)	63.14%	4.27%	(15.60%)	36.05%
2	832,830	(22,186,000)	53.17%	7.55%	(2.72%)	46.53%
3	4,677,980	(20,420,390)	43.90%	8.68%	1.60%	57.50%
4	8,410,455	(19,036,020)	36.40%	9.21%	3.66%	67.08%
5	12,034,150	(17,984,000)	30.58%	9.49%	4.78%	74.49%

Conclusion

The treatment of input variables as being static in nature is common in DCF analysis. The financial literature, recognizing a weakness, suggests the use of sensitivity analysis. This process is characterized by making arbitrary changes to input parameters so as to reflect a normal, above-normal, and below-normal situation. This paper suggests a supplementary methodology as it conducts an analysis by allowing ten unique input parameters to randomly fluctuate in order to see the resulting impact on three major output items, the net present value, the modified internal rate of return, and the profitability index. This research conducted a Monte Carlo Simulation, incorporating these input-output variables. The resulting sensitivity analysis was based on 50,000 iterations creating stochastic output probability distributions. The financial literature is extended as this was done under an American Put Option Strategy where these iterations were conducted for each year in a five-year time horizon. The premise of this strategy is realistic as it offers an investor a real option. The investor has the right but not the obligation to sell off the asset at any time during its economic life. The histogram in Figure 2 and Figure 3 exemplify a more robust sensitivity analysis revealing the minimum, maximum, and mean NPV values. It also reports on volatility of values with the standard deviation of the distribution. This paper produced a Tornado Chart (Figure 1) showing the impact of each of the input variables on the NPV in each year of the project's life and ranked them as to their level of influence on the outputs (Table 11). Data Trends revealed that the capitalization rate and asking price on the lease were consistently ranked one and two from years two through five of the Put Strategy.

The going-in cap rate had the highest impact on the NPV in each year from time period two through time period five. In year one, with its high probability of a negative NPV, credit loss and vacancy loss were ranked two and three, respectively. By year five, the impact of vacancy loss had slipped to seventh place while the impact of credit loss stood in fifth place.

The credit loss, expense stop, and maintenance were mixed in their impact as they changed places among the four, five, six, and seven slots. The selling costs, insurance expense, and property taxes were ranked at the bottom in four of the five years.

Setting itself apart from a static study, the paper's use of a stochastic model and the Monte Carlo enables an investor to determine the probability of the annual NPV being negative and whether the MIRR exceeds the project's capitalization rate. Under the American Put Option Strategy, the option to sell Project LaSalle during year one would not be exercised as the mean NPV is negative and is supported by the 63.14% probability of a negative value and a 36% chance that the MIRR would be higher than the capitalization rate. An interesting trend emerges from Table 9 and Table 12. In each year of the project's economic life, the lower 5% percentile revealed negative NPVs. In each year, the lower 5% percentile showed that the MIRR was below the capitalization rate. Further, while the mean NPV was positive for years three, four, and five, the distributions revealed probabilities of negative cash flows of 44%, 36% and 31% respectively. Contrast this with a static analysis where the mean positive NPVs over this same time period would constitute acceptable years in which to exercise the option to sell off the capital asset. The resulting probability and lower percentile distributions support this paper's premise that if a strategy is followed based on a static analysis and that adheres to strict NPV and MIRR rules, it would totally ignore the possibility of future annual net operating income cash flows being inadequate to cover the initial cost of Project LaSalle.

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